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Faculty of Economics and Social Sciences

Department of Economics

The Role of Sectoral Shifts in the Great Moderation

Daniel Burren

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Schanzeneckstrasse 1
Postfach 8573
CH-3001 Bern, Switzerland
http://www.vwi.unibe.ch

The Role of Sectoral Shifts in the Great Moderation*

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January 17, 2008

Abstract

In this paper, I study the drop of real GDP volatility which has been observed in the United States during the postwar period.

This paper thoroughly estimates how much sectoral shifts contributed to this phenomenon called the Great Moderation.

In a short section, Stock and Watson (2003) find that this contribution is negligible, however, their data is disaggregated only up to 10 sectors. Blanchard and Simon (2001) come to the same result. Using a new estimation method and more disaggregated data, I find that sectoral shifts contributed between 15% and 30% to the great moderation. Moreover, I find that if in the year 1949 sectoral shares had been equal to what they were in 2005, then the conditional and unconditional standard deviation of GDP growth would have been, on average, 20-25% lower in the postwar period. Finally, I find that the shift out of durable goods production has significantly stabilized real GDP growth.

As a methodological contribution, I show how to use the particle filter to estimate latent covariance matrices when they follow a Wishart autoregressive process of order one.

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†University of Bern and Study Center Gerzensee, danielburren@gmail.com

I use this in order to get, for each observation period, an estimation of the covariance matrix of the sectoral growth rates. Since real GDP growth is the sum of these sectoral growth rates weighted by the sectoral shares, it is then straightforward to use these covariance matrices to express the conditional variance of GDP growth in each period as a function of sectoral shares. Computing the unconditional variance of GDP growth as a function of sectoral shares is a bit more involved, but also quite easy using Monte Carlo simulations.

My methodology to estimate covariance matrices is preferable to alternatives like estimating a multivariate GARCH model or using a Nadaraya-Watson estimator for the following reasons: The multivariate GARCH model has undesirable properties for the Monte Carlo simulations and involves estimating a large number of parameters. The Nadaraya-Watson estimator, on the other hand, does not guarantee to give positive definite covariance matrices due to the limited number of observations available for estimating the relatively big covariance matrices.

Keywords: Great moderation, Sectoral Shifts, Stochastic Volatility, Wishart Autoregressive Process, Particle Filter, ARCH-GARCH, Bayesian Estimation.

JEL-Classification: C11, C32, E32.

Contents

1 Introduction

In this paper, I study the drop of real GDP volatility which has been observed in the United States during the postwar period.

There is a rich body of competing explanations for this phenomenon called the Great Moderation. The contribution of this paper is to thoroughly assess the part of the Great Modration which can be attributed to sectoral shifts.

In a short section, Stock and Watson (2003) estimate that this contribution is negligible, however, their data is disaggregated only up to 10 sectors. Blanchard and Simon (2001) come to the same result. Using more disaggregated data, I find that sectoral shifts contributed between 15% and 30% to the great moderation, depending on the chosen method. Moreover, I find that if in the year 1949 sectoral shares had been equal to what they were in 2005, then the conditional and unconditional standard deviation of GDP growth would have been, on average, 20-25% lower in the postwar period. Finally, I find that the shift out of durable goods production has significantly stabilized real GDP growth.

As a methodological contribution, I show how to use the particle filter to estimate latent covariance matrices when they follow a Wishart autoregressive process of order one. I use this in order to get, for each observation period, an estimation of the covariance matrix of the sectoral growth rates. Since real GDP growth is the sum of these sectoral growth rates weighted by the sectoral shares, it is then straightforward to use these covariance matrices to express the conditional variance of GDP growth in each period as a function of sectoral shares. Computing the unconditional variance of GDP growth as a function of sectoral shares is a bit more involved, but also quite easy using Monte Carlo simulations.

My methodology to estimate covariance matrices is preferable to alternatives like estimating a multivariate GARCH model or using a Nadaray-Watson estimator for the following reasons: The multivariate GARCH model has undesirable properties for the Monte Carlo simulations and involves estimating a large number of parameters. The Nadaraya-Watson estimator, on the other hand, does not guarantee to give positive definite covariance matrices due to the limited number of observations available for estimating the relatively big covariance matrices.

In the first part of the paper, I look at aggregated real GDP growth only. Using GARCH specifications for the innovation variance in a univariate autoregressive (AR) process for GDP growth, I show that there is evidence for a non-linear decreasing trend in the conditional GDP variance. I then argue that this non-linear trend could have been caused by sectoral shifts.[1]

In the second part, I analyze explicitly the role of sectoral shares in US-GDP volatility using GDP by industry data from the NAICS two digits dataset (Bureau of Economic Analysis). I proceed as follows.

First, I estimate AR(1) processes for the sectoral growth rates and check the stability of (i) the regression parameters, using the test of Andrews and Ploberger (1994), as well as of (ii) the innovation variances, using the cumulative sum of squares test proposed by Inclán and Tiao (1994). Interestingly, I find that the innovation variance in the AR(1)s of the sectors mining, utilities and information increased. Only for durable goods, I find a significant decrease in the innovation variance. This goes against the hypothesis that good luck (i.e. less frequent economic shocks) is at the source of the great moderation.

Then, I use the estimated AR(1) processes for Monte Carlo simulations which I make in order to assess the implication of sectoral shifts for GDP volatility. In a first set of simulations, I draw the innovation errors from a multivariate normal distribution with constant covariance matrix. Since the covariance matrix is constant, changes in the volatility of simulated GDP (obtained by aggregating simulated sectoral production) must come from changes in the shares. Depending on whether the pre-84 or the post-84 sample covariance matrix of the AR(1) residuals is used for the simulations, I find that sectoral shifts explain 31.7%, respectively 14.4% of the great moderation.

In the second set of simulations, I assume that the covariance matrix of the AR(1)-innovations is time varying and changes according to a Wishart

[1] This contributes also to the discussion whether it is a trend decrease or whether it is a one-time drop in post-war GDP volatility.

autoregressive process of order one. I use the particle filter to draw from the posterior distribution of the covariance matrices. The results show that the standard deviation of GDP would have been 25.6% lower in the period 1949-1984 and 16.4% lower in the period 1984-2005 if the sectoral shares had been already in 1949 equal to what they were in 2005.

Besides simulating unconditional standard deviations, I use the filtered sequence of covariance matrices to express the conditional standard deviation of GDP growth in every period as a function of sectoral shares. I find that if the sectoral shares had been equal to what they were in 2005 during the entire postwar period, then the conditional standard deviation would have been, on average, 24.7% lower to what it would have been if the shares had been equal to their 1949 values during the entire period. I argue therefore that sectoral shifts made GDP easier to forecast.

The paper is structured as follows. Section 2 analyzes the conditional volatility of GDP growth using ARCH-GARCH methods. Section 3 studies the dynamics of sectoral growth rates and presents the first set of Monte Carlo simulations. Section 4 introduces the Wishart process and the particle filter and shows how conditional and unconditional GDP volatility depend on sectoral shares. Section 5 concludes.

2 GDP Growth and Volatility Revisited

Fang and Miller (2006) and Blanchard and Simon (2001) estimate univariate ARCH-GARCH processes for real GDP and its volatility and both find that the Great Moderation is best explained by a decrease in the innovation variance of GDP growth whereas they do not find breaks in its mean and first order autocorrelation. They disagree, however, on the nature of this decrease: Fang and Miller (2006) find a one time drop in the unconditional GDP growth variance, which they date in the first quarter 1982 (1982:1), whereas Blanchard and Simon (2001) claim that it is a trend decline which was temporarily interrupted in the 1970s and early 1980s.

In this chapter, I shortly review this discussion and give evidence in favor of the trend decline hypothesis. I show that a quadratic time trend in the conditional variance equation can not be rejected as easily as a linear trend when a dummy for a one time break in the early 1980s is included. The conjecture that a part of the Great Moderation is caused by sectoral shifts even calls for a non-linear trend because GDP variance is a quadratic function in the sectoral shares which can be seen as follows. Denote by y_t real GDP and by v_t^i the value added in sector i. Since $y_t = \sum_i v_t^i$, it is straightforward to get

$$\begin{aligned} \frac{\Delta y_t}{y_{t-1}} &= \sum_i \tau_{t-1}^i x_t^i \\ \Rightarrow Var_t\left(\frac{\Delta y_t}{y_{t-1}}\right) &= \sum_i (\tau_{t-1}^i)^2 Var(x_t^i) + \sum\sum_{i \neq l} \tau_{t-1}^i \tau_{t-1}^l Cov(x_t^i, x_t^l) \end{aligned}$$

where $x_t^i = \Delta v_t^i / v_{t-1}^i$ and $\tau_t^i = v_t^i / y_t$. Hence, linear trends in the shares translate into a quadratic trend for the conditional GDP growth variance (and standard deviation).

Since the shares are bounded between zero and one, it is even likely that already trends in shares are non-linear, implying a polynomial of order higher than two for the time trend in GDP growth variance. However, if one includes such higher order terms in the estimation, one finds coefficients that are close to zero.

Instead of estimating a polynomial, one can also approximate a non-linear trend by a piecewise linear one with changing coefficient. Doing this, I find a significant negative coefficient for the period prior to 1972:1 and non-significant coefficients for the periods afterwards.

In the following subsections, the results of Fang and Miller (2006) are first presented and then extended estimations with non-linear trend specifications are done. The conclusions remain the same with quarterly and annual GDP growth.

2.1 Linear Trend versus Break

Fang and Miller (2006) find that the volatility of quarterly GDP growth in the postwar period is well modeled by either a GARCH or an ARCH model where a dummy variable is included in order to capture the Great Moderation. Using a procedure based on an iterated cumulative sums of squares (ICSS) algorithm (this algorithm allows to detect multiple points of variance change), proposed by Inclán and Tiao (1994), they identify one and only one change in volatility at 1982:2 and therefore allow the intercept in the variance equation to change in 1982:1. Adding a time trend to these volatility models does not improve significantly the fit. Concerning the serial correlation of GDP growth rates, they find that it is well captured by an AR(2) model, written as

$$y_t \quad = \quad a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t.$$

For the innovation variance, they assume once a GARCH(1,1), written as

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \omega\, t + \gamma\, d_t,$$

and once an ARCH(2), written as

$$\alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \omega\, t + \gamma\, d_t,$$

where y_t is the growth rate, t a time trend, d_t a dummy variable which equals 1 for $t \geq 1982:1$. In their original specification, they also tested for GARCH-in-mean effects by including σ in the AR(2) equation. Since σ proved insignificant, I do not include it in my estimations in order to gain efficiency. The estimations are summarized in Table 1.

Table 1: Linear Trend and Break 1982:1, Period 1947-2006

GARCH Results

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2
0.4763***	0.2651***	0.1673**
(0.0770)	(0.0687)	(0.0716)

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \omega\ t + \gamma\ d_t$

α_0	α_1	β_1	ω	γ
0.4089*	0.0155	0.6188***	-0.0001	-0.3290**
(0.2265)	(0.0688)	(0.1898)	(0.0005)	(0.1745)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
2.1781	5.21766	0.9390	4.6395	0.6590
[0.536]	[0.521]	[0.816]	[0.591]	[0.719]

ARCH Results

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2
0.4777***	0.2903***	0.1486**
(0.0805)	(0.0735)	(0.0718)

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 + \omega\ t + \gamma\ d_t$

α_0	α_1	α_2	ω	γ
1.0494***	0.1705*	0.0964	-0.0007	-0.7443***
(0.2129)	(0.0873)	(0.1064)	(0.0011)	(0.2077)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
2.0795	5.0338	0.8096	3.8642	0.8167
[0.556]	[0.539]	[0.847]	[0.695]	[0.665]

Standard errors are in parentheses, p-values in brackets; $LBQ(k)$ and $LBQ^2(k)$ are the Ljung-Box Q-statistics for the standardized residuals and the squared standardized residuals testing for autocorrelation up to k lags. *, ** and *** denote 10, 5 and 1 % significance level.

None of the Ljung-Box Q-statistics[2] is significant which is evidence for

[2] Q-statistic $= T(T+2)\sum_{j=1}^{k} \frac{corr(e_t, e_{t-j})}{n-j} \xrightarrow{H_0} \chi_k^2$ where H_0 is absence of serial correlation.

absence of remaining autocorrelation in the standardized residuals and the squared standardized residuals. The Jarque-Bera[3] test indicates that the standardized residuals are normal giving support for the model.

The insignificant time trend is contrary to what find Blanchard and Simon (2001). Based on a panel regression with the G7 countries, they argue that it was the increased inflation inflation volatility which temporarily took output volatility off its trend decline in the 1970s and early 1980s. In order to test this hypothesis, I add to the variance regression a dummy variable which is one from 1973:1[4] until 1981:4 (the results are similar when the dummy is one until 1983:4); I denote it by d_t^{70s}. The result, however, is the same as before, the linear trend is not significant in neither the ARCH nor the GARCH specification. The same is true for the d^{70s}-dummy, which goes against the claims of Blanchard and Simon (2001). The one time only break hypothesis cannot be rejected. In order to make the model parsimonious, and the estimates more precise, I continue using an ARCH(1) specification only. The Jarque-Bera test and Ljung-Box statistics indicate that this model is still rich enough to capture the volatility dynamics. The results of this model are in Tables 2 whereas the results of a model with ARCH(2) and one with GARCH(1,1) specification can be found in the appendix in Table 15. The reader can verify that the findings remain the same.

[3]Jarque-Bera $= \frac{T}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \overset{H_0}{\rightarrow} \chi^2_2$ where S is the sample skewness and K the sample kurtosis.

[4]The oil embargo of the Arab nations against the countries supporting Israel in the Yom Kippur War took place in 1973 which caused the first oil-price shock

Table 2: Linear Trend, Break 1982:1 and 70s Dummy, Period 1947-2006

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2		
0.4588***	0.2740***	0.1724**		
(0.0748)	(0.0747)	(0.0609)		

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \omega\, t + \gamma_1\, d_t + \gamma_2\, d_t^{70s}$

α_0	α_1	ω	γ_1	γ_2
0.9875***	0.2282***	-0.0011	−0.5916**	0.6236
(0.2033)	(0.0982)	(0.0013)	(0.2680)	(0.4105)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
2.1331	6.0040	0.9630	3.9274	0.6607
[0.545]	[0.423]	[0.810]	[0.687]	[0.719]

Standard errors are in parentheses, p-values in brackets; $LBQ(k)$ and $LBQ^2(k)$ are the Ljung-Box Q-statistics for the standardized residuals and the squared standardized residuals testing for autocorrelation up to k lags. *, ** and *** denote 10, 5 and 1 % significance level.

2.2 Non-linear Trend versus Break

The conclusions are different if a quadratic trend-term is included in the variance equation. The coefficients of the trend polynomial and of the dummy for the period 1973:1-1981:4, d_t^{70s}, are now significant (at the 10% level) whereas the dummy for post 1982:1 period no longer is (the conclusions are the same if 1984:1 is taken as the break point; the corresponding results are in the appendix in Table 16). This is supportive for the argument of Blanchard and Simon (2001) stating that the 1970s were special (the results are in Table 3). Within a GARCH(1,1) or a ARCH(2) model, the trend coefficients are even significant at the 1% level (the corresponding results are in Table 17 in the appendix).

Table 3: GDP - Quadratic Trend in Volatility, Period 1947-2006

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2
0.4659***	0.2644***	0.1774***
(0.0754)	(0.07344)	(0.06242)

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \omega_1\, t + \omega_2\, t^2 + \gamma_1\, d_t + \gamma_2\, d_t^{70s}$

α_0	α_1	ω_1	ω_2	γ_1	γ_2
1.4578***	0.2084**	−0.0120*	0.00003*	−0.1069	1.0330***
(0.4312)	(0.0982)	(0.0073)	(0.00002)	(0.3637)	(0.4564)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
2.0013	5.7139	0.6396	3.3593	0.4992
[0.572]	[0.456]	[0.887]	[0.763]	[0.779]

Standard errors are in parentheses, p-values in brackets; $LBQ(k)$ and $LBQ^2(k)$ are the Ljung-Box Q-statistics for the standardized residuals and the squared standardized residuals testing for autocorrelation up to k lags. *, ** and *** denote 10, 5 and 1 % significance level.

Finally, I estimated an ARCH model with a piecewise linear trend in GDP volatility. It can be summarized by the following equations.

$$\begin{aligned} y_t &= a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \omega_1\, t + \omega_2\, (t * d_t^{70s}) + \omega_3\, (t * d_t) + \gamma\, d_t \end{aligned}$$

In this model, the dummy for the 1970s, d_t^{70s}, is omitted. According to the Q-statistics and the Jarque-Bera test (reported in Table 4), also this model is rich enough to capture excess kurtosis of GDP volatility. One can see that there was a significant negative trend in GDP volatility in the pre-1972 period. A Wald test reveals that afterwards, no significant trend remains (I tested the hypotheses $H_1 : \omega_1 + \omega_2 = 0$ and $H_2 : \omega_1 + \omega_3 = 0$ and cannot reject one of them; the respective p-values are 0.88 and 0.57). If one estimates the model with the 1970s-dummy, one finds that this dummy is not significant and that its inclusion only lowers the p-value of the trend coefficient ω_1, giving even more support for the trend. At the same time, the 1982 - dummy d_t is significant again.

The lack of a significant time trend in the second half of my sample goes in no way against my conjecture, that a part of the decline in GDP volatility is caused by sectoral shifts. It could just be evidence that the growing service sector first diversified the economy and therefore made GDP growth more stable; but that eventually these stabilization gains from diversification became smaller. It is even conceivable that in the future GDP volatility starts to increase when the service sector continues to grow, thus making the economy less diversified again. Figure 1 shows the evolution of the shares of private service-producing and private goods-producing industries.

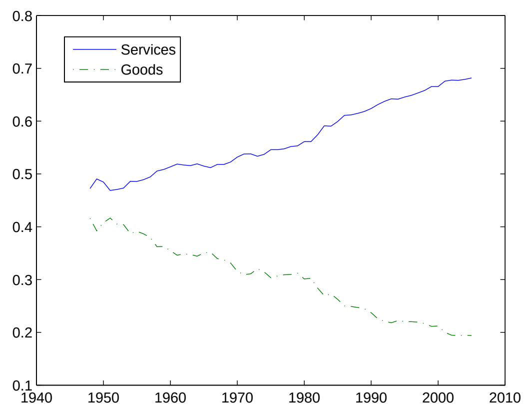

Figure 1: Shares of Goods and Service Sector

Table 4: GDP - Piecewise Linear Trend in Volatility, Period 1947-2006

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2
0.4581***	0.2684***	0.1806***
(0.0738)	(0.0733)	(0.0619)

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \omega_1\, t + \omega_2\, (t * d_t^{70s}) + \omega_3\, (t * d_t) + \gamma\, d_t$

α_0	α_1	ω_1	ω_2	ω_3	γ
1.4244***	0.2227**	−0.0093*	0.0100***	0.0086*	−1.1123***
(0.4094)	(0.0991)	(0.0053)	(0.0041)	(0.0054)	(0.4559)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
1.8853	5.7425	0.5838	3.3685	0.6907
[0.597]	[0.453]	[0.900]	[0.761]	[0.708]

Standard errors are in parentheses, p-values in brackets; $LBQ(k)$ and $LBQ^2(k)$ are the Ljung-Box Q-statistics for the standardised residuals and the squared standardised residuals testing for autocorrelation up to k lags. *, ** and *** denote 10, 5 and 1 % significance level.

2.3 Annual Data

Since the sectoral series, which I use for the analysis of the sectoral shifts, are annual and not quarterly, I do a short analysis for annual GDP growth as well (I use annual GDP from BEA and deflate it with the implicit GDP deflator[5] and population then I calculate discrete time growth rates: $100 \times (\text{GDP}_t\text{-GDP}_{t-1})/\text{GDP}_{t-1}$). First, I estimate an AR(1) process and check whether one can find significant breaks in the innovation variance using also the ICSS algorithm described in Inclán and Tiao (1994). In order to get the correct asymptotic distribution of the test statistic under the null of no break in the conditional variance, it is important that the residuals are serially uncorrelated. A Fischer test reveals there are no significant partial autocorrelations in the residuals up to two lags.[6] I find one and only one

[5] The implicit GDP deflator is from the BEA and given by the ratio of the GDP current-dollar value to its chained-dollar value.

[6] The p slope coefficients of an AR(p) are estimates of the partial autocorrelations up to lag p. Since I suspect heteroskedastic errors, I use robust standard errors as proposed

break point in 1984 (the results are summarized in Table 5). This is in line with existing literature: Stock and Watson (2003), McConnell and Perez-Quiros (2000) and Andrews and Ploberger (1994) also detect a unique break in GDP growth variance in 1984. It suggests estimating a ARCH model with a dummy in the variance equation that equals zero before 1984 and one from 1984 on.

The results in the first panel of Table 6 reveal a significant quadratic time trend, giving support for the idea that sectoral shifts have an influence on GDP volatility. The Jarque-Bera test does not reject normality and the Q-statistics do not detect remaining correlation in standardized and squared standardized residuals. This is evidence that the estimated ARCH model is rich enough to describe GDP growth and volatility. Figure 3 shows a graph of the ARCH implied conditional standard deviation for this model.

If the quadratic trend is replaced by a linear one with changing coefficients, the results are less clear because no coefficient is significant. I suspect that this is due to the small number of observations in the annual dataset (57 yearly growth rates for 1949-2005 whereas with the quarterly data set 240 observations for 1947:1-2006:4 are available). (Figure 2 shows real per capita GDP growth for 1948-2005).

Table 5: AR(1) - Annual GDP Growth, Period 1948-2005

AR(1) equation: $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$

a_0	a_1	R^2	F-test	Break Date(s)
1.8539***	0.1117	0.01	0.0453	1984
(0.4841)	(0.1572)		[0.956]	

White's robust standard errors are in parentheses, p-values in brackets; F-test is for joint significance of $\varepsilon-$AR(2) coefficients. *, ** and *** denote 10, 5 and 1 % significance level.

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by White (1980). This potential heteroskedasticity is also the reason why Box-Pierce and Ljung-Box tests are inappropriate.

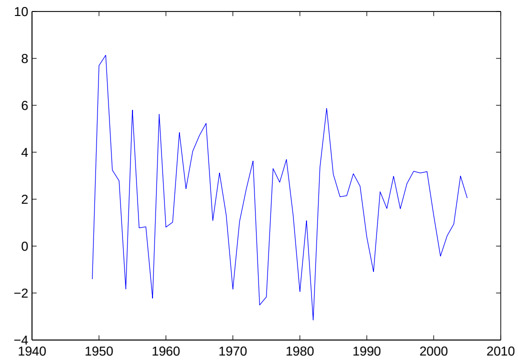


Figure 2: Real GDP growth rate for the US, 1948-2005

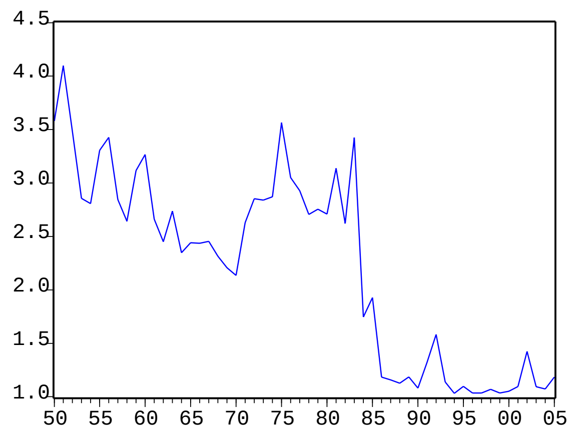


Figure 3: Conditional standard deviation with a quadratic trend

Table 6: Annual GDP - ARCH with Non-Linear Trend, Period 1948-2005

Quadratic Trend

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$

a_0	a_1
2.0219***	0.2762**
(0.4404)	(0.1506)

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \omega_1\ t + \omega_2\ t^2 + \gamma_1\ d_t + \gamma_2\ d_t^{70s}$

α_0	α_1	ω_1	ω_2	γ_1	γ_2
9.2599***	0.2015	−0.2968***	0.0030***	−1.0123	4.1049
(3.3731)	(0.2673)	(0.0401)	(0.0001)	(3.3098)	(4.9508)

Test statistics

$LBQ(2)$	$LBQ^2(2)$	Jarque-Bera
0.2161	1.3333	1.8763
[0.642]	[0.248]	[0.391]

Trend with changing coefficients

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$

a_0	a_1
2.0079***	0.2979*
(0.4191)	(0.1611)

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \omega_1\ t + \omega_2\ (t * d_t^{70s}) + \omega_3\ (t * d_t) + \gamma\ d_t$

α_0	α_1	ω_1	ω_2	ω_3	γ
9.8500	0.1986	−0.2976	0.2215	0.2829	−8.0961
(8.7045)	(0.2750)	(0.4503)	(0.2401)	(0.4542)	(9.1925)

Test statistics

$LBQ(2)$	$LBQ^2(2)$	Jarque-Bera
0.3014	1.7013	1.7589
[0.583]	[0.192]	[0.415]

Standard errors are in parentheses, p-values in brackets; $LBQ(k)$ and $LBQ^2(k)$ are the Ljung-Box Q-statistics for the standardized residuals and the squared standardized residuals testing for autocorrelation up to k lags. *, ** and *** denote 10, 5 and 1 % significance level.

3 Sectoral Growth and Volatility

3.1 The Data

The data used are the two digits annual US-GDP by industry series from the Gross-Domestic-Product-by-Industry Accounts (Bureau of Economic Analysis) ranging from 1948 to 2005. These series sum up to nominal GDP. They were deflated using the implicit GDP deflator[7] and expressed in per capita terms using the Civilian non-institutional Population index from the Bureau of Labor Statistics. The dataset consists of the following 22 sectors (in brackets are the abbreviations): Agriculture - forestry - fishing - hunting (Agriculture), mining (Mining), utilities (Utilities), construction (Construction), durable goods (Durables), nondurable goods (NonDurables), wholesale trade (Wholesale), retail trade (Retail), transportation & warehousing (Transp-Ware), information (Information), finance & insurance (FinInsur), real estate - rental & leasing (ReEstatRent), professional - scientific & technical services (ServScieTech), management of companies & enterprises (ManagComp), administrative & waste management services (ManagAdminWaste), education services (ServEduc), health care & social assistance (ServicesHealth), arts - entertainment & recreation (Recreat), accommodation & food services (Serv-FoodAcc), other services without government (ServOth), federal government (GovFed) and state & local government (GovLocal).

The correlation coefficient between the averages and the standard deviations of the sectoral growth rates (both computed over the entire time period) is -0.47. This is reflected in Figure 4 where a scatter plot of the mean growth rates and the coefficient of variation[8] of the series is shown. It means that the stable sectors have grown faster than the volatile ones. The aim of the remaining part of the paper is to find out, how much of the Great Moderation is due to this growth and volatility pattern.

[7]The implicit GDP deflator is from the BEA and given by the ratio of the GDP current-dollar value to its chained-dollar value.

[8]The coefficient of variation is a normalization of the standard deviation and is obtained by dividing the standard deviation with the mean of the serie. I took the absolute values of the means because agriculture had negative average growth.

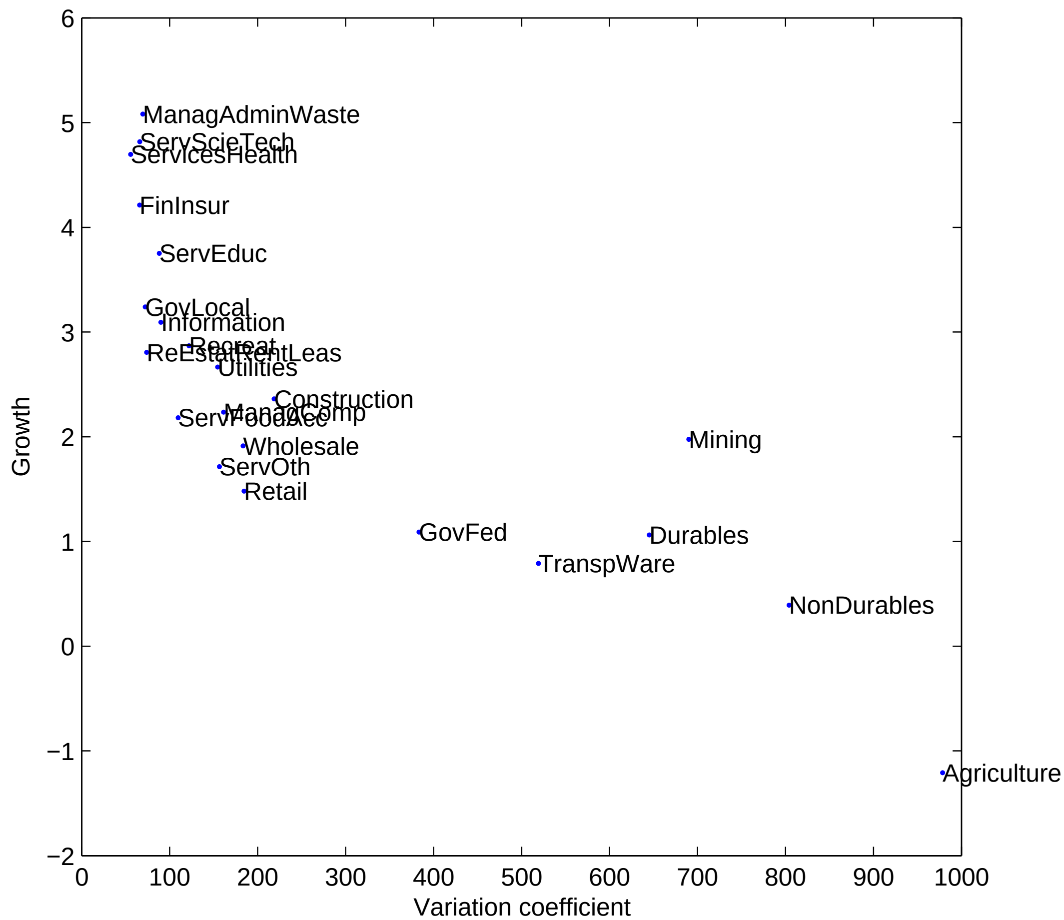


Figure 4: Growth and Coefficient of Variation

3.2 Dynamics of Sectoral Growth and its Volatility

In this section, I look for a process which can describe the dynamics of sectoral growth rates. In a first step, for each sectoral growth rate, denoted by x_t^i, an AR(1) is estimated. Then, parameter stability is analyzed and, in a second step, it is tested whether there are breaks in the innovation variance. The aim is to get models which can be used to simulate sectoral growth (this is done in the subsequent sections).

The AR(1) for sector i is written as

$$x_t^i = c^i + \rho^i x_{t-1}^i + \varepsilon_t^i. \tag{1}$$

The estimations of this process are in Table 7. It shows that, with the exception of agriculture, there is either positive or insignificant autocorrelation in the growth rates. Almost all service related sectors have a significant positive constant, which implies that they have a positive mean growth rate. This is not the case for the non-service sectors agriculture, mining, and durable goods production. Such a finding reflects sectoral shifts. The reported F-statistics (F-stat) come from testing whether two additional lags should be included in the AR regressions. They indicate that there is significant partial autocorrelation up to lag two left in the residuals of agriculture, health care & social assistance and federal government.[9]

[9] F-stat test the joint significance of a_1^i and a_2^i in

$$\varepsilon_t^i = a_0^i + a_1^i \varepsilon_{t-1}^i + a_2^i \varepsilon_{t-2}^i + \nu_t^i$$

where instead of Newey-West robust standard errors White (1980)'s robust standard errors are used because under the null, there is no serial correlation left, however, there could still be potential heteroskedasticity.

Table 7: AR(1) for sectoral growth rates

Sectors	c^i	$\sigma(c^i)$	ρ^i	$\sigma(\rho^i)$	F-stat	R^2
Agriculture	-1.09	1.47	-0.25^{**}	0.11	3.32	0.06
Mining	1.91	1.79	0.22^{**}	0.09	2.22	0.05
Utilities	1.72^{**}	0.73	0.27	0.17	1.63	0.08
Construction	1.44^{*}	0.82	0.41^{***}	0.2	0.36	0.17
Durables	1.08	0.92	0.03	0.15	0.33	0.001
Nondurables	0.48	0.42	-0.02	0.13	0.51	0.0004
Wholesale	1.99^{***}	0.61	0.01	0.10	1.78	0.0000
Retail	1.21^{***}	0.42	0.19	0.13	0.10	0.03
TranspWare	0.87	0.57	0.01	0.12	0.07	0.0001
Information	2.83^{***}	0.59	0.06	0.14	0.02	0.004
FinInsur	2.85^{***}	0.54	0.29^{***}	0.10	0.41	0.10
ReEstRentLeas	1.33^{***}	0.34	0.50^{***}	0.10	0.69	0.26
ServScieTech	3.10^{***}	0.78	0.36^{***}	0.13	0.46	0.13
ManagComp	2.24^{***}	0.58	0.02	0.13	0.02	0.0004
ManagAdminWaste	3.39^{***}	0.74	0.33^{***}	0.11	0.0011	0.11
ServEduc	1.26^{**}	0.56	0.65^{***}	0.10	0.17	0.42
ServHealth	4.61^{***}	1.26	0.02	0.23	4.07	0.0003
Recreat	2.82^{***}	0.59	0.03	0.12	0.17	0.0008
ServFoodAcc	1.88^{***}	0.35	0.14	0.11	0.05	0.02
ServOth	1.27^{***}	0.47	0.25^{*}	0.14	0.25	0.06
GovFed	0.75	0.60	0.31^{**}	0.13	28.16	0.10
GovLocal	0.52^{**}	0.26	0.80^{***}	0.08	1.10	0.69

Newey-West robust standard errors (computed with one lag) are reported. *, ** and *** denote 10, 5 and 1 % significance level. The theoretical 95th quantile of the F-distribution for testing significance of autocorrelation in residuals is 3.18.

The remaining autocorrelation could come from structural changes in parameters. Andrews and Ploberger (1994) proposed a procedure to test the null of constant regression parameters against the alternative of a single unknown break time. This test can be implemented as follows. For each growth rate and for each integer τ for which $[T * 0.15] \leq \tau \leq T - [T * 0.15]$, the following regression equation is estimated with ordinary least squares (T is the sample size and $[x]$ returns the rounded down part of x).

$$x_t^i = (1 - d_t(\tau))c_1^i + d_t(\tau)c_2^i + (1 - d(\tau)_t)\rho_1^i x_{t-1}^i + d_t(\tau)\rho_1^i x_{t-1}^i + \varepsilon_t^i \qquad (2)$$

$d_t(\tau)$ is a dummy variable which is equal to zero for all $t < \tau$ and equal to one for $t \geq \tau$. This dummy variable divides the data in two subsamples where c_p and ρ_p for $p = 1, 2$ are the coefficients of the AR(1) equation (1) restricted to subsample p. Potential variations in the variance of ε_t^i are accounted of by using White's robust standard errors. The advantage of writing the test regression in this way is that covariances between estimators in different subsamples are easily computed. The Wald statistic is then given by

$$W(\tau) = \begin{pmatrix} c_1 - c_2 \\ \rho_1 - \rho_2 \end{pmatrix}' \begin{pmatrix} Var(c_1 - c_2) & Cov(c_1 - c_2, \rho_1 - \rho_2) \\ Cov(c_1 - c_2, \rho_1 - \rho_2) & Var(\rho_1 - \rho_2) \end{pmatrix}^{-1} \begin{pmatrix} c_1 - c_2 \\ \rho_1 - \rho_2 \end{pmatrix}$$

where the apostrophe stands for transpose. Under the null of constant parameters, the biggest $W(\tau)$, given by $\xi_W = \sup_\tau(W(\tau) : [T * 0.15] \leq \tau \leq T - [T * 0.15])$, converges to a Brownian bridge. The τ for which $W(\tau)$ is maximized, is an estimate of the break period. Using the critical values provided by Andrews and Ploberger (1994), I find significant breaks (at the 5% level) for mining, utilities, retail trade, real estate - rental & leasing, professional - scientific & technical services, education services, health care & social assistance, accommodation & food services and state & local government but none for agriculture and federal government (the results are in Table 2). Therefore, it seems that the remaining autocorrelation in these two latter sectors, detected in the regression over the entire sample, can not be attributed to parameter changes. For health care & social assistance, however, the F-statistic for testing remaining autocorrelation in residuals dropped from 4.18 (Table 1) to 0.17 (Table 8) and is no longer significant at the 5% level.

I conclude that for all sectors, except agriculture and federal government, an AR(1), if necessary with changing coefficients, is sufficient to capture the autocorrelation in yearly growth rates.

Table 8: Andrews Parameter Stability Test

Sectors	$\hat{c}_1^i$	$\hat{c}_2^i$	$\hat{\rho}_1^i$	$\hat{\rho}_2^i$	$\hat{\mu}_1$	$\hat{\mu}_2$	max W	F-stat	τ
Mining	2.18	1.02	0.30	0.13	3.11	1.17	10.11	2.29	'56
	(1.81)	(3.35)	(0.17)	(0.16)	(5.69)	(3.96)			
Utilities	3.73	1.71	0.33	0.09	5.58	1.88	17.74	0.32	'87
	(2.15)	(0.79)	(0.20)	(0.20)	(10.39)	(1.17)			
Retail	1.35	0.81	0.15	0.35	1.60	1.25	13.23	0.06	'60
	(0.64)	(0.56)	(0.17)	(0.27)	(0.83)	(0.50)			
ReEstat-	1.24	2.93	0.56	-0.31	2.79	2.23	20.46	0.21	'86
RentLeas	(0.47)	(0.91)	(0.12)	(0.31)	(0.82)	(1.54)			
ServScie-	8.18	2.95	-0.21	0.35	6.75	4.55	24.95	0.36	'67
Tech	(1.60)	(0.76)	(0.23)	(0.13)	(9.57)	(2.78)			
ServEduc	1.18	3.01	0.67	0.21	3.54	3.79	20.58	0.22	'77
	(0.59)	(1.21)	(0.10)	(0.25)	(1.46)	(3.51)			
ServHealth	9.21	2.25	-0.55	0.43	5.96	3.97	26.35	0.17	'92
	(1.64)	(0.65)	(0.22)	(0.16)	(8.96)	(1.67)			
ServFood-	2.25	1.83	-0.73	0.26	1.30	2.46	22.43	0.01	'62
Acc	(0.60)	(0.38)	(0.23)	(0.07)	(0.68)	(0.87)			
GovLocal	0.73	1.02	0.77	0.36	3.23	1.60	64.42	1.01	'56
	(0.39)	(0.46)	(0.09)	(0.24)	(0.75)	(0.28)			

White's robust standard deviations are in parentheses. *max W* is the maximum Wald statistic from Andrews test; the 5% critical value is 8.68. The F-stat comes from testing significance of remaining autocorrelation in residuals up to two lags; its theoretical 95th quantile is 3.18. τ is an estimate of the break period.
The estimated mean growth rate for sector i is calculated as $\hat{\mu}^i = \frac{c^i}{1-\rho^i}$ and its standard deviation is obtained using the delta method. The delta method consists of doing a first order Taylor expansion of $\hat{\mu}^i$ in $\hat{c}^i$ and $\hat{\rho}^i$ and then using the covariance matrix of the parameter estimators.

In order to get an accurate stochastic model for sectoral growth rates, I also need an idea of how their innovation variances evolved.

According to proponents of the good luck theory, economic fluctuations have moderated thanks to less volatile shocks. If this is true, then there should be breaks in the innovation variances of sectoral growth rates, i.e. the variances of the ε_t^is should have decreased. This is tested using the ICSS test of Inclán and Tiao (1994) on the residuals of the AR(1) regression where

(1) is replaced by (2) for the series for which Andrews test detects breaks in parameters. The results give evidence for one-time breaks in the variances of the residuals of mining, utilities, durable goods and information with break dates 1973, 1973, 1959 and 1995.[10] In Table 9 are estimates of the conditional standard deviations before and after the break date. Also reported are the unconditional standard deviations of the respective growth rates, x_t^i.

Table 9: Variance Breaks

Sectors	Break Date τ	$\sigma(\varepsilon^i_{t<\tau})$	$\sigma(\varepsilon^i_{t\geq\tau})$	$\sigma(x^i_{t<\tau})$	$\sigma(x^i_{t\geq\tau})$
Mining	1973	5.66	16.47	5.92	16.84
Utilities	1973	2.10	4.19	3.46	4.14
Durables	1959	11.22	5.74	9.50	6.02
Information	1995	2.30	4.32	2.36	4.15

Interestingly, the innovation variances in mining, utilities and information increased. This goes against the wisdom that GDP growth has become more stable thanks to less volatile economic shocks.

Comparing these results with the results of the parameter break tests in Table 8, it can be seen that there is no clear relationship between parameter breaks and conditional variance breaks. Only for mining and utilities there is evidence for breaks in both, parameters and innovation variance; the break periods, however, lie several years apart. This extends the GARCH-in-mean analyzes of Fang and Miller (2006) to sectoral data. Similar to their findings for real GDP growth, I do not find (G)ARCH-in-mean-like effects for sectoral data.[11]

Inclán and Tiao (1994) point out that their test is not sure to detect small variance changes in series of 100 observations or less. Therefore, I

[10]The ICSS test also indicates significant volatility breaks for agriculture (the one period forecast standard deviation increased from 5.90 to 13.75 in 1972) and federal government (the conditional standard deviation decreased from 14.70 to 2.13 in 1953). However, since there is significant autocorrelation in these residuals, an assumption for the application of the ICSS test is not satisfied and therefore, the results are not reliable.

[11]I do not report explicit GARCH-in-mean estimations because for many sectors, I did not find GARCH specifications which produce homoskedastic standardized residuals.

assume now that the potential break date is 1984. If the Good Luck theory is true, according to which GDP volatility decreased thanks to absence of large shocks, then the innovation variances of the sectoral growth rates should have decreased at the same time as GDP growth variance (which the ICSS test finds to be in 1984 for annual GDP growth). The test statistic is given by

$$F = \frac{\hat{\sigma}^2(\varepsilon^i_{t<1984})}{\hat{\sigma}^2(\varepsilon_{t\geq 1984})^i}$$

where ε^i_t is the residual from regression (1), respectively (2). It follows, under the null of equal variances, a F-distribution with $n_1 - K$ and $n_2 - K$ degrees of freedom (n_i is the number of observations in subsample i and K the number of regressors). The alternative hypothesis is, in this case, a decrease in sectoral innovation variance.

The test finds significant decreases in the variances for durable goods, nondurable goods, retail trade, transportation & warehousing and state & local government (the results are in Table 10).[12]

I conclude that there are changes in the innovation variances. In order to take them into account, I model the covariance matrix of sectoral growth rates explicitly in section 4 using a Wishart specification. This has two advantages over letting the univariate innovation variances change according to what the break tests indicated. First, it takes into account potential changes in the covariances of the AR(1)-innovations of different sectors. Second, it does not require to take a stand on break dates. However, before introducing this new methodology, I present the results of some simple Monte Carlo simulations in the next subsection.

[12] It also indicates a significant decrease in federal government variance, however, this could be spurious since there is significant correlation left in these residuals.

Table 10: Standard Deviations - Break Date 1984

Sectors	$\sigma(\varepsilon^i_{t<1984})$	$\sigma(\varepsilon^i_{t\geq 1984})$	$\sigma(x^i_{t<1984})$	$\sigma(x^i_{t\geq 1984})$	F-stat
Agriculture	11.42	11.21	11.09	13.12	1.04
Mining	11.31	16.03	11.61	16.25	0.50
Utilities	3.71	2.91	4.36	3.11	1.63
Construction	5.09	4.27	5.37	4.87	1.42
Durables	8.03	4.67	8.04	4.61	2.95*
Nondurables	3.62	2.17	3.66	2.27	2.79*
Wholesale	3.81	2.94	3.92	2.87	1.68
Retail	3.12	1.90	2.85	2.54	2.70*
TranspWare	4.72	2.84	4.68	2.92	2.76*
Information	2.36	3.34	2.33	3.41	0.50
FinInsur	2.56	2.35	3.02	2.40	1.18
ReEstRentLeas	1.83	1.63	2.26	1.77	1.60
ServScieTech	2.81	3.15	2.82	3.71	0.80
ManagComp	3.72	3.51	3.64	3.55	1.12
ManagAdminWaste	3.12	3.77	3.06	4.18	0.69
ServEduc	2.80	2.01	4.00	1.95	1.96
ServHealth	2.17	2.10	2.62	2.36	1.06
Recreat	3.06	3.99	3.00	3.90	0.59
ServFoodAcc	2.40	1.89	2.59	2.05	1.61
ServOth	2.87	2.17	2.73	2.45	1.74
GovFed	4.85	2.19	5.06	2.35	4.89*
GovLocal	1.44	0.80	2.65	1.08	3.26*

The theoretical 95th quantile of the F-distribution for testing equality in ε^i_t-variance is 2.04.

3.3 Monte Carlo Simulations with counter-factual Covariance Matrices

In order to get a first idea of the importance of sectoral shifts, I divided the data into two subsamples, one with the data 1948-1983 and one with the data 1984-2005. Assuming that each sectoral growth rate follows an autoregression as described in equation (1) respectively (2) (i.e. coefficients which were found to change are allowed to do so), GDP growth is simulated 100'000 times for each subsample (agriculture and federal government is omitted because the AR(1) representation was found to be insufficient).

In a first set of simulations, the covariance matrix of the AR-innovations ε_t^i is allowed to change in 1984 and assumed to be constant otherwise (this assumption is relaxed in the Section 4 where the covariance matrix is allowed to change according to a Wishart transition.). In a second set, it is assumed to equal its pre-84 sample analogue throughout the entire sample whereas in a third set of simulations, it is set to its post-84 sample analogue for both subsamples. Then, for each set of simulations, the average (median) GDP standard deviation is computed. The purpose is to get an idea of what GDP volatility would have been with different underlying sectoral covariance matrices but with the same average sectoral growth. This isolates the influences of sectoral share movements on GDP volatility from the influences coming from changes in the covariance matrix.

The first set of simulations yields an average (median) GDP volatility of 2.93 (2.92) in the first and 1.55 (1.51) in the second period. For 95% of the simulations, the standard deviations are inside the interval $[2.21, 3.76]$ in the first subsample and inside $[0.99, 2.35]$ in the second subsample. Observed GDP volatility (without agriculture and federal government), which is 3.02 pre-84 and 1.49 post-84, lies inside the respective confidence interval. With the second simulation, where only the covariance matrix of the first period is used, I find that average (median) standard deviation decreased from 2.93 (2.92) in the first period to 2.49 (2.45) in the second period. The ratio of this difference over the difference from the previous simulation (where the average fell from 2.93 to 1.55) equals 31.9% (33.3%). It can be interpreted as a measure for the part of the Great Moderation which is due to sectoral shifts. The remaining 68.2 % can be attributed to the changes in the covariance matrix. We might compute different contributions if instead of the covariance matrix of period 1, the one of period 2 is used. I find an average (median) standard deviation of 1.80 (1.78) for period 1 and one of 1.60 (1.54) for period 2. The ratio of the differences is now 14.5% (17.0%). Table 11 shows the results of these simulations.

In the appendix, results can be found for three additional sets of simulations. In the first one, agriculture and federal government are included. Using averages (medians), the estimated part of the moderation due to sectoral shifts is 33.9% (33.3%) with the covariance matrix of period 1 and 13.9% (24.4%) with the covariance matrix of period 2. In the second set of simulations, all sectors are included but the coefficients in the AR(1) processes are not allowed to change. This might be an incorrect description for some of the sectors, but on the other hand, there are no effects of coefficient-breaks on simulated GDP volatility. With the covariance matrix of the first period, I find that sectoral shifts explain 34.5% (33.3%) whereas with the covariance matrix of the second period they explain 21.8% (25.0%). The most pessimistic results are received with the third simulation where only sectors are included for which an AR(1) with constant coefficients fully captures serial correlation (giving a sample with 11 of the 22 sectors). The share attributed to sectoral shifts is 19.5% (20.6%), respectively 6.3% (6.7%).

Table 11: Simulation Results

Simulation 1: Different Covariances in Period 1 and Period 2

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
2.93 (2.92)	$[2.21, 3.76]$	1.55 (1.51)	$[0.99, 2.35]$

Simulation 2: Covariances of Period 1 in both Periods

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
2.93 (2.92)	$[2.21, 3.75]$	2.49 (2.45)	$[1.62, 3.61]$

Simulation 3: Covariances of Period 2 in both Periods

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
1.80 (1.78)	$[1.36, 2.35]$	1.60 (1.54)	$[1.02, 2.58]$

4 Wishart Specification

In this section the latent covariance matrix of the sectoral growth rates are assumed to follow a Wishart autoregressive process of order one (henceforth WAR(1)). Conditional on their covariance matrix, the processes of the sectoral growth rates are assumed to be Gaussian AR(1). Thereby, the breaks in the coefficients detected by the ICSS test in the previous section are considered. This can be summarized in the following state-space system.

$$x_t | W_t \sim N(c_t + \rho_t x_{t-1}, W_t) \tag{3}$$

$$W_t | W_{t-1} \sim WAR(1) \tag{4}$$

The observed sectoral growth rates x_t^i are stacked in vector x_t, the vector c_t has elements c_t^i and ρ_t is a diagonal matrix with elements ρ_t^i.

The above state-space system provides a structure which allows to estimate the sequence of W_ts. I use the filtered covariance matrices (W_t) for two purposes. The first one is to infer how the conditional standard deviation of GDP growth depends on sectoral shares whereas the second one is to infer how the unconditional standard deviation depends on sectoral shares. While the first purpose is straight forward to do given the W_ts, the second one requires Monte Carlo simulations due to the non-linearity of the problem.

This way of estimating the covariance matrices (W_t) of the sectoral growth rates can be viewed as a Bayesian estimation where the WAR(1) is the prior distribution of the covariance matrices. This procedure is simply a multivariate extension of the univariate case where the prior of a standard deviation is assumed to be a Gamma distribution. This interpretation then justifies calibrating the parameters of the WAR(1) instead of estimating them, just as one would calibrate the parameters of the Gamma prior in the univariate case.

Before the implementation and the results are presented, however, I discuss the Wishart distribution and my motivation to use it.[13]

[13] A detailed discussion of WAR(1) processes can be found in Gourieroux, Jasiak, and Sufana (2005). The following paragraphs are based on their paper.

Intuitively, the Wishart process (W_t) is an autoregressive process for covariance matrices of dimension n_s. Its dynamics is specified by the moment generating function which is defined by

$$M_{W_{t+1}|W_t}(\Gamma) = \mathbb{E}_t[\exp(Tr(\Gamma W_{t+1}))]$$

where $\mathbb{E}_t$ is the conditional expectation on information up to time t, Γ is a deterministic and symmetric matrix of real numbers and Tr denotes the trace operator. Since W_{t+1} is symmetric, we can write

$$Tr(\Gamma W_{t+1}) = \sum_{i=1}^{n}\sum_{l=1}^{n}(\Gamma)_{il}(W_{t+1})_{il}.$$

The explicit moment generating function of a WAR(1) is given by

$$\begin{aligned} M_{W_{t+1}|W_t}(\Gamma) &= \mathbb{E}_t[\exp(Tr(\Gamma W_{t+1}))] \\ &= \frac{\exp(Tr[M'\Gamma(I_{n_s} - 2\Sigma\Gamma)^{-1}MW_t])}{[\det(I_{n_s} - 2\Sigma\Gamma)]^{K/2}} \end{aligned}$$

where K is a scalar degree of freedom strictly larger than $n-1$, I_{n_s} the identity matrix of dimension n_s, M the $n_s \times n_s$ matrix of autoregressive parameters and Σ is a $n_s \times n_s$ symmetric, positive definite matrix. The moment generating function is defined for matrices Γ which satisfy $\| 2\Sigma\Gamma \| < 1$, where $\| \cdot \|$ is the operator norm.[14]

The beauty of this process is, that it gives positive definite matrices. For

[14]The norm of a matrix A of dimension $r \times c$ (or, more precisely, of a linear operator $A : \mathbb{R}^c \to \mathbb{R}^r$), is defined to be the supremum of all numbers $|Ax|$ where $|\cdot|$ is any norm in $\mathbb{R}^r$ and where x ranges over all vectors in $\mathbb{R}^c$ with $|x| \leq 1$.

If A is symmetric it can be written as PDP' where P is an orthogonal eigenvector matrix and D a diagonal eigenvalue matrix. With the Euclidean norm, we get

$$|Ax| = (x'A'Ax)^{(1/2)} = (x'P'D^2Px)^{(1/2)} = (y'D^2y)^{(1/2)}$$

and since $|y| = |Px|$ the maximization can be done over y. We see that $|Ax|$ is maximal if the element in y corresponding to the biggest absolute eigenvalue in D is 1. Hence, for a symmetric matrix, the norm corresponds to the biggest absolute eigenvalue. Since $\| \Sigma\Gamma \| \leq \| \Sigma \| \| \Gamma \|$, the condition is satisfied if the product of the two largest eigenvalues is less than 0.5.

integer K, it can be interpreted as follows:

$$W_t = \sum_{k=1}^{K} z_{k,t} z'_{k,t}$$

where the processes $(z_{kt}), k = 1, \ldots, K$ are independent vector processes, each of length n_s, that satisfy

$$z_{k,t} = M z_{k,t-1} + e_{k,t} \quad e_{k,t} \sim N(0, \Sigma)$$

If $K = 1$, $M = 0$ and $\Sigma = 1$, one can recognize the $\chi^2(1)$ distribution (Chi-squared with one degree of freedom) as a special case of the Wishart distribution.[15]

The stochastic matrix W_t is of full rank with probability one if the degree of freedom, denoted by K, is equal to or greater than n_s. This can be seen with the following argument. If $K = 1$, then each column in W_t is a multiple of the vector $z_{1,t}$. Therefore, the columns of W_t span a space of dimension 1 which means that the matrix is singular if $n_s > 1$. If $K = 2$, then column i of W_t is a linear combination given by $z_{1,t}(i) z_{1,t} + z_{2,t}(i) z_{2,t}$ where $z_{k,t}(i)$ is the ith element of vector $z_{k,t}$. Since the $z_{k,t}$-vectors are continuously distributed, they are almost surely different[16] and therefore, the columns of W_t span a room of dimension 2. Continuing the argument, one finds that the dimension of the column-space equals $\min(K, n_s)$.

It is interesting to see how W_t behaves when K increases. Obviously, if the covariance matrix Σ of the $e_{k,t}$ vectors is not decreasing in K, then W_t has an exploding second moment matrix and therefore it does not converge to a constant real matrix. Hence, the calculation must be done for $\Sigma = K^{-1}\tilde{\Sigma}$ where $\tilde{\Sigma}$ is a constant matrix. Setting $\tilde{z}_{k,t} := \sqrt{K} z_{k,t}$ allows to write

$$W_t = \frac{1}{K} \sum_{k=1}^{K} \tilde{z}_{k,t} \tilde{z}'_{k,t} \tag{5}$$

[15] It is however not correct to say that the Wishart distribution is a multivariate χ^2-distribution because off-diagonal elements of W_t have positive probability mass on negative values.

[16] *almost surely* means *with probability one*

where

$$\tilde{z}_{k,t} = M\tilde{z}_{k,t-1} + \tilde{e}_{k,t} \quad \tilde{e}_{k,t} \sim N(0, \tilde{\Sigma}) \tag{6}$$

If all the eigenvalues of M are strictly inside the unit circle, the unconditional distribution of $\tilde{z}_{k,t}$ is $N(0, \tilde{\Sigma}(\infty))$ where $\tilde{\Sigma}(\infty)$ solves

$$\tilde{\Sigma}(\infty) = M\tilde{\Sigma}(\infty)M' + \tilde{\Sigma}.$$

Hence, $\{\tilde{z}_{k,t}\tilde{z}'_{k,t}\}_{k=1}^{K}$ is a sample of identically and independently distributed random variables having finite second moment matrices. This enables the application of the strong law of large numbers according to which the sum (5) converges almost surely to $\mathbb{E}[\tilde{z}_{k,t}\tilde{z}'_{k,t}]$. Therefore, in the limit, the WAR(1) process is a degenerated process with constant matrices as its realizations.

The degree of freedom K can be fractional. However, in this case, the interpretation of the VAR(1) processes $(z_{k,t})$ is no longer valid. For the univariate case, this corresponds to generalizing the χ^2-distribution to the Gamma-distribution.

It is important to understand that the Wishart specification is different to the concept of multivariate generalized ARCH (MGARCH)[17] in the following senses.

(i) In a MGARCH model, the volatility of x_{t+1} conditional on information up to time t depends on past realizations which is not the case for the state-space system with the Wishart specification. Technically speaking, for the MGARCH model the conditional volatility in time $t+1$ is measurable with respect to the Borel algebra generated by $(x_i)_{i=1}^{t}$ (it is said to be predictible) whereas for the Wishart specification it is measurable with respect to the Borel algebra generated by the innovations of the WAR(1) up to $t+1$. Consequently, the MGARCH is has undesirable properties for the intended simulations.

(ii) Compared to the Wishart distribution, the MGARCH cannot be interpreted as a prior distribution for the volatility matrix. While it is,

[17] Engle and Kroner (1993) and Tse and Tsui (2000) discuss MGARCH models.

within a Bayesian framework, justified to use a fully calibrated Wishart distribution as a prior for filtering conditional volatility matrices, the estimation of volatility matrices in a MGARCH framework requires estimating a great number of parameters. Even the multivariate ARCH(1) model, given by

$$vech(\Sigma_t) = b + A\ vech(x_t x_t')$$

involves $\left(\frac{n_s(n_s+1)}{2}\right)^2 + \frac{n_s(n_s+1)}{2}$ parameters (n_s is, in my case, the number of sectors).[18] The literature has proposed to put restrictions on the parameters (see for example Engle and Kroner (1993)) which however can be *"complicated and hard to interpret"*[19] because they have to be such that the conditional volatility matrices are positive definite.

For a discussion of further differences between MGARCH and WAR(1), I refer to Gourieroux, Jasiak, and Sufana (2005) and the literature cited therein.

Compared to a nonparametric covariance matrix estimator, like the Nadaraya -Watson estimator, the Wishart specification has the advantage of yielding positive definite covariance matrices. The Nadaraya-Watson estimator is a weighted average of the cross-product matrices of the AR-residuals (denoted by ε_t), i.e. it is of the from

$$W_t^{NW} = \frac{\sum_\tau \omega\left(\frac{\tau - t}{b}\right) e_t e_t'}{\sum_\tau \omega\left(\frac{\tau - t}{b}\right)}.$$

Since I want to filter a covariance matrix of dimension 19,[20] it gives only positive definite matrices if the range over which these averages are computed is at least 19 years long. Given that I have observations for only 58 years, I prefer not using it.

[18]The operator $vech(.)$ stacks the different elements in Σ; since there are $n_e := \frac{n_s(n_s+1)}{2}$ such different elements, there are n_e equations each involving n_e regressors which gives the n_e^2 parameters in A.

[19]see Gourieroux, Jasiak, and Sufana (2005)

[20]Since the AR(1) process is not rich enough to capture the autocorrelation in agriculture and federal government, these two sectors are omitted. In order to consider the private economy only, the sector state & local government is also omitted. This leaves 19 sectors in the dataset.

4.1 Particle Filter

The observation equation (3) and the transition equation (4) form a non linear state space system. Therefore, the particle filter, also known as sequential Monte Carlo filter, can be used to draw from the posterior distribution of the covariances W_t given the observed growth rates up to time t. This posterior distribution is denoted by $p(W_t|x_{1:t}, \theta)$ where $x_{1:t}$ are the observations up to time t and $\theta = \{M, \Gamma, \Sigma, K\}$. The sample average of these draws is an estimation of $\mathbb{E}[W_t|x_{1:t}]$, which is the minimum mean squared error estimator for the conditional covariance matrix. In the following, $f_x(x_t|W_t)$ is the density of x_t given W_t (and also given $c_t + \rho_t x_{t-1}$). The transition density of the WAR(1) is written as $f_w(W_t|W_{t-1}, \theta)$. The procedure to be implemented has three steps.

Step 1, Initialization Draw N times from the unconditional distribution of W_0, $p(W_0|\theta)$. This gives a sample of N particles which is designed by $\{w^i_{0|0}\}^N_{i=1}$.

Step 2, Prediction For each of the particles in the sample from the previous step, given by $\{w^i_{t-1,t-1}\}^N_{i=1}$, generate one draw from $f_w(W_t|w^i_{t-1,t-1}, \theta)$. The result is a sample of N particles, $\{w^i_{t|t-1}\}^N_{i=1}$ that are drawn from $p(W_t|x_{1:t-1}, \theta)$. This follows from

$$\begin{aligned}
p(W_t|x_{1:t-1}, \theta) &= \int p(W_t, W_{t-1}|x_{1:t-1}, \theta) dW_{t-1} \\
&= \int f_w(W_t|W_{t-1}, \theta) p(W_{t-1}|x_{1:t-1}, \theta) dW_{t-1} \\
&= \mathbb{E}_{W_{t-1}}[f_w(W_t|W_{t-1}, \theta)|x_{1:t-1}, \theta] \\
&\approx \frac{1}{N} \sum_{i=1}^{N} f_w(W_t|w^i_{t-1,t-1}, \theta)
\end{aligned}$$

where $\mathbb{E}_{W_{t-1}}$ is the expectation taken over $W_t - 1$. The Markov property allows to replace $f_w(W_t|W_{t-1}, x_{1:t-1}, \theta)$ by $f_w(W_t|W_{t-1}, \theta)$ in the second equation. Intuitively, the density $p(W_t|x_{1:t-1})$ can be approximated by a mixture of conditional transition densities where the probability that we draw from $f_w(W_t|w^i_{t-1,t-1}, \theta)$ equals $1/N$ for each i. This uniform probability mass distribution justifies drawing once from each $f_w(W_t|w^i_{t-1,t-1},\theta)$ to get a sample from $p(W_t|x_{1:t-1},\theta)$.

Step 3, Updating Draw, with replacement, N times from the previous sample, $\{w^i_{t|t-1}\}^N_{i=1}$. The probability to draw $w^i_{t|t-1}$ is given by the so called normalized importance weight, denoted by π^i_t, which is calculated according to:

$$\pi^i_t = \frac{f_x(x_t|w^i_{t|t-1})}{\sum_{i=1}^N f_x(x_t|w^i_{t|t-1})}$$

The resulting sample is a draw from the discretized density $p(W_t|x_{1:t},\theta)$ which can be seen from the following argument.

According to Bayes rule, it is

$$p(W_t|x_{1:t},\theta) = \frac{f_x(x_t|W_t)p(W_t|x_{1:t-1},\theta)}{f_x(x_t|x_{1:t-1})}$$

where the denominator can be approximated by

$$\begin{aligned} f_x(x_t|x_{1:t-1}) &= \int f_x(x_t|W_t)p(W_t|x_{1:t-1},\theta)dW_t \\ &= \mathbb{E}_{W_t}[f_x(x_t|W_t)|x_{1:t-1},\theta] \\ &\approx \frac{1}{N}\sum_{i=1}^N f_x(x_t|w^i_{t|t-1}) \end{aligned}$$

I abuse notation and write

$$\mathbb{P}(W_t = w^i_{t|t-1}|x_{1:t},\theta) \approx \underbrace{\frac{f_x(x_t|W_t)}{\sum_{i=1}^N f_x(x_t|w^i_{t|t-1})}}_{=\pi^i_t} \mathbb{P}(W_t = w^i_{t|t-1}|x_{1:t-1},\theta)$$

where $\mathbb{P}$ is a probability measure. In words, I express it as follows. Since, in Step 2, each element in $\{w^i_{t|t-1}\}^N_{i=1}$ is drawn from $p(W_t|x_{1:t-1},\theta)$, if the i'th element is drawn with probability π^i_t, the overall probability mass of this element is approximately $p(w^i_{t|t-1}|x_{1:t},\theta)$.
The drawn sample is referred to by $\{w^i_{t|t}\}^N_{i=1}$ and is the input to Step 2 for the next iteration.

The minimum mean squared error estimate of the latent covariance matrix at time t is given by

$$\frac{1}{N}\sum_{i=1}^{N} w_{t|t}^{i}$$

A detailed discussion of particle filtering methods can be found in Arulampalam, Maskell, Gordon, and Clapp (2002). These methods have been used by Sungbae (2005) and by Fernandez-Villaverde and Rubio-Ramirez (2006) to estimate a second order approximated DSGE model.

4.2 Implementation - A Bayesian Estimation

I estimate the latent covariances assuming that their prior distribution is a fully calibrated WAR(1) process. This procedure can be considered as the multivariate extension of a Bayesian estimation of a standard deviation where a Gamma-prior is assumed for the standard deviation. Posterior draws of the covariance matrix are then filtered using the observed sectoral growth rates in the updating step.

The parameters that I have to choose are the elements of the matrices M and Σ and the degree of freedom K. For simplicity, I assume M to be diagonal with identical elements $\sqrt{\lambda}$ where $\lambda = 0.8$.[21] The matrix $K\Sigma$ is set equal to the sample covariance matrix of the data multiplied by $(1-\lambda)$ and K equal to 19.

I justify the calibration of $K\Sigma$ by the discussion in the previous section according to which W_t converges to $\tilde{\Sigma}(\infty)$. If $\tilde{\Sigma}(\infty)$ is assumed to be close to the sample covariance matrix, the calibration follows from equation (6) which implies $K\Sigma = \tilde{\Sigma} = (1-\lambda)\Sigma(\tilde{\infty})$. The calibration of K is justified by the number of sectors in my data set, which is 19[22] ($K = 19$ is thus the minimal degree of freedom that gives regular W_t matrices).

Due to limited computer capacity, I run the particle filter 20 times drawing 20'000 particles each time. Then, I compute the mean of the posterior means

[21] I have also tried $\lambda = 0.2$ and $\lambda = 0.5$. The conclusions regarding the contribution of sectoral shares in GDP volatility remain the same. A higher λ means that the Wishart is more persistent and has weaker innovations.

[22] Agriculture, federal government and state & local government are left out.

and take this as an estimation for the standard deviation.[23] Since the covariance matrices, which I want to filter, are quite big, it is well possible that even 100'000 particles are not enough to get precise estimates. Therefore, instead of drawing the 19 initial $z_{k,0}$s from their unconditional distribution, I set them equal to the residual vectors from the AR(1) estimations (i.e. to $x_t - c_t - \rho_t x_{t-1}$) using the residual vectors of the first 9 years twice. The filtered covariance matrix in the first period, W_1, is therefore based on $z_{k,1}$s which should not be too far away from the $z_{k,1}$s obtained with enough particles.

4.3 Results

As a first assessment of the econometric model, one can compare the results in Table 9 in section 3.2 (which contains the results of the ICSS variance break tests) with Figures 11, 12, 13 and 14 in the appendix. These figures show that the filtered AR(1) innovation standard deviations of mining, utilities, durable goods and information square well with the findings of the ICSS variance break test.[24]

4.3.1 Conditional GDP Volatility

I use the filtered covariance matrices to assess how the conditional standard deviation of GDP growth depends on sectoral shares. Formally, since GDP growth is given by

$$\frac{\Delta y_t}{y_{t-1}} = \sum_i \tau_{t-1}^i x_t^i$$

the conditional variance of GDP growth as a function of sectoral shares is

$$Var_{t-1}\left(\frac{\Delta y_t}{y_{t-1}}\right) = \tau_{t-1}' W_t \tau_{t-1} \tag{7}$$

[23] I have some evidence that the algorithm's covering of the domain is not too bad: when I run the algorithm ten times drawing 10'000 particles each time, I find approximately the same results.

[24] Graphs of filtered standard deviations of other series as well as graphs of filtered covariances are available from the author upon request.

where τ_t is a column vector with the shares τ_{t-1}^i of sectoral production in real GDP. The conditional standard deviation is just the square root of this expression. It is plotted in Figure 5 for the period 1949-2006. We can see that it decreased from above 3.5 to below 2. It looks similar to the conditional standard deviation obtained with the ARCH model in Section 2 and plotted in Figure 3. However, a difference is, that the standard deviation in Figure 5 is smoother than the one obtained with the ARCH model. This reflects that the variance obtained with the Wishart specification is more robust to extreme realizations of squared GDP growth rates.

In order to compare the two models explicitly, I regress the conditional variance, obtained by equation (7), on the following variables: a constant, the residual from the univariate GDP growth AR(1), a time trend, a squared time trend, a dummy variable for the post-84 period and a dummy variable for the '70s. This can be written as:

$$Var_{t-1}\left(\frac{\Delta y_t}{y_{t-1}}\right) = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \omega_1 t + \omega_2 t^2 + \gamma_1 d_t + \gamma_2 dt^{70s} + v_t,$$

where v_t is the error term. This corresponds to the ARCH model estimated in Section 2. The results of both estimations are reported in Table 12. It can be seen that compared to the ARCH models, all coefficients in the OLS regression are significant. The coefficients of the trend components are, in absolute value, slightly larger in the OLS equation than in the ARCH equation. The constants are of similar size.

Table 12: Annual ARCH with Quadratic Trend, Period 1948-2005

OLS Results

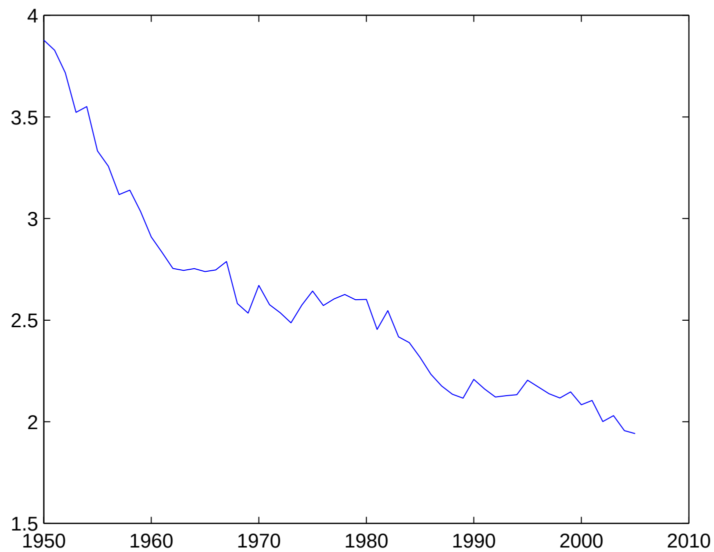
α_0	α_1	ω_1	ω_2	γ_1	γ_2
13.3607	0.0244	-0.4159	0.0044	0.8540	1.4023
(0.3728)	(0.0104)	(0.0286)	(0.0005)	(0.4721)	(0.3456)

Results from Section 2

α_0	α_1	ω_1	ω_2	γ_1	γ_2
9.2599	0.2015	-0.2968	0.0030	-1.0123	4.1049
(3.3731)	(0.2673)	(0.0401)	(0.0001)	(3.3098)	(4.9508)

Standard errors are in parentheses.

Figure 5: Conditional GDP Standard Deviation



A big advantage of computing the GDP volatility using equation (7) over using an ARCH model, is that changes in the GDP volatility can be related to changes in the GDP composition. This is done next.

In order to get an idea of how GDP volatility is influenced by its composition, I compute three counter-factual paths for the conditional GDP standard deviations. The first one is obtained by evaluating equation 7 in each period at the shares of 1949. The second and third one are obtained by evaluating the same equation at the shares of 1977, respectively 2005. While the shares are kept constant when computing the GDP volatility paths, the same time-varying W_t is used for all three paths. Hence, the three paths differ only by the underlying assumed GDP composition. Their evolutions are plotted in Figure 6. The top graph corresponds to the shares of year 1949, the middle one to the shares of year 1977 and the bottom one to shares of year 2005. On average, the conditional standard deviations with 2005-shares are 24.7% (19.1%) lower than the ones with 1949-shares (1977-shares).

The difference remains important if instead of end of period shares, average shares are used. In Figure 7, the upper graph is the counter-factual conditional GDP standard deviation where sectoral shares are set to their pre-84 averages whereas the lower graph is the conditional GDP standard deviation with sectoral shares set to their post-84 averages. The standard deviation with post-84 shares is, on average, 17.7% lower than the standard deviation with pre-84 shares.

This suggests that an important part of the drop in the conditional GDP standard deviation can be explained by sectoral shifts. The findings are the same if a less persistent WAR(1) is assumed. The results of two alternative calibrations are in the appendix in Figures 15 and 16.

Figure 6: Conditional GDP Standard Deviation with given constant shares

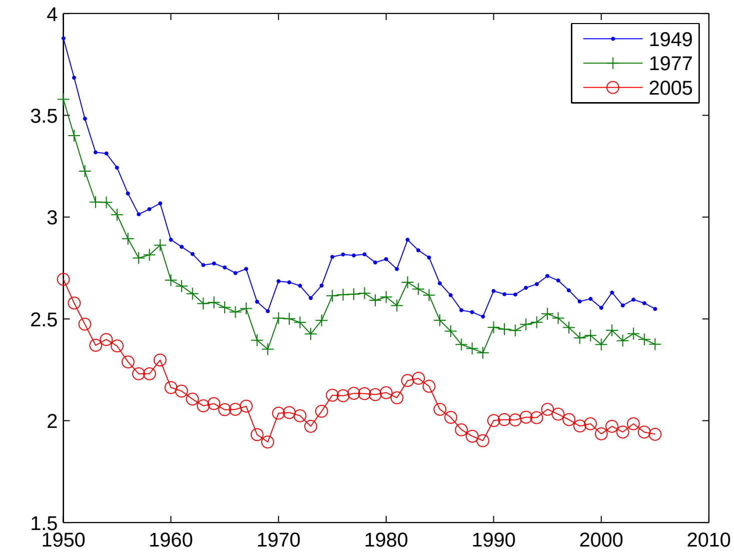
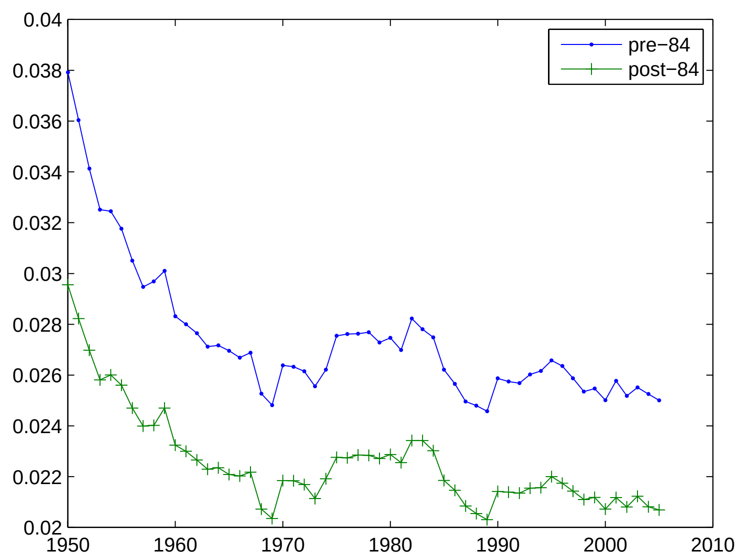


Figure 7: Conditional GDP Standard Deviation with average shares



4.3.2 Unconditional GDP Volatility

Computing the unconditional GDP volatility analytically as a function of initial (1949) sectoral shares is very hard. Therefore, I simulate it for the post-war period in the following way. I set initial shares and then I simulate sectoral productions using the AR(1)s as estimated in Section 3.2 (taking into account the parameter breaks) where the innovations are drawn from multivariate normals with covariance matrices equal to the filtered W_ts.[25] I further compute the standard deviation of the implied GDP growth rate for the pre-84 and the post-84 period. I repeat this 100'000 times and finally compute the average standard deviations.

The resulting average standard deviations, when initial shares are equal to what they were in 1949, 1977 and 2005, are reported in Table 13. The 1949-share simulation can be used to test my specification. GDP volatility in the data is 3.21 in the pre-84 and 1.63 in the post-84 period. These numbers are inside the intervals $[2.06, 3.24]$ and $[1.38, 2.62]$ which contain 90% of the simulated standard deviations of the pre-84, respectively of the post-84 period.[26]

Table 13: Simulated GDP standard deviation

Initial Shares	1950-1984	1985-2005
1949	2.62	1.95
1977	2.37	1.75
2005	1.95	1.63

We can see that the average simulated standard deviations depend on the chosen initial sectoral shares. The simulated standard deviation for the 2005 (1977) initial shares is 25.6% (17.7%) lower in the pre-84 period than the

[25] Since the AR(1) parameters are different, sectors grow at different rates in my simulations, and therefore, shares depart from their initial values. I did not choose to calibrate the AR(1) parameters such that shares stay, on average, close to their initial values. The reason is, that there are many different possible calibrations which can do this and which have, by themselves, different implications on simulated GDP volatility. In my view, this makes the results hard to interpret.

[26] These intervals are symmetric, i.e. obtained using the 5% and 95% quantiles.

simulated standard deviation for the 1949 initial shares. For the post-84 period, the respective percentage is 16.4% (10.3%).[27]

4.3.3 Relevant Sectoral Shifts

In the previous sections, I have found that GDP volatility depends, to a non-negligible extend, on its decomposition. The purpose of this section is to determine the relevant sectoral changes behind this relationship.

The five most important sectoral shifts - in absolute value and in order of decreasing importance - took place in the sectors durable goods, nondurable goods production, finance & insurance, professional - scientific and technical services and in the sector health care & social assistance.[28] The numbers in Table 14 show that both durable and nondurable goods production have become less important than the three service sectors finance and insurance, professional - scientific and technical services and health care & social assistance. In the years 2001-2005, these three service sectors made up over 20% of real GDP.

The question is, whether these shifts have had a relevant impact on real GDP volatility. In order to answer this, the evolutions of the conditional standard deviations of all five sectors are plotted in Figure 8. We can see that in every period, the standard deviations of the two production sectors are bigger than the standard deviations of the three service sectors. This suggests that indeed, these five shifts have had a dampening effect on GDP volatility.

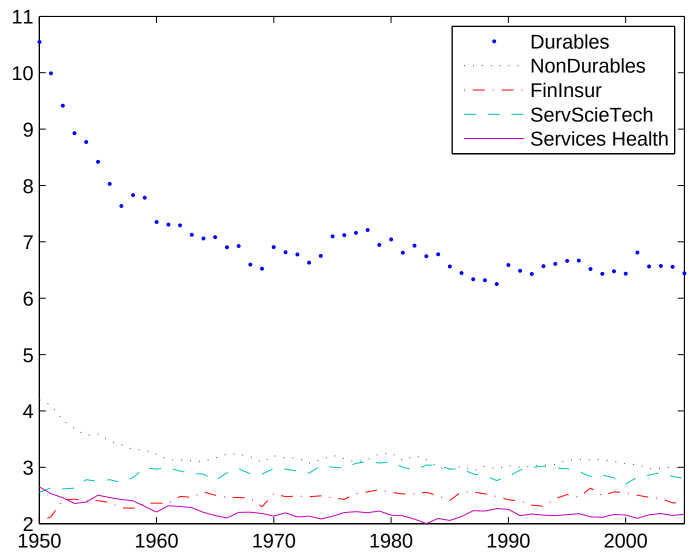
[27]The results do barely change with different Wishart calibrations: For $\lambda = 0.2$, simulated standard deviation for the 2005 (1977) initial shares is 24.4% (9.6%) lower in the pre-84 period and 17.5% (10.9%) in the post-84 period. For $\lambda = 0.5$, the respective percentages are 24.7% (9.6%) and 17.7% (11.1%).

[28]The absolute difference of the average shares in the years 1948-1952 and the average shares in the years 2001-2005 is computed. The list hardly changes if, instead, the absolute difference between pre-84 and post-84 average shares is taken. The only difference is, that finance and insurance experiences now the least important change of these five sectors.

Table 14: Average shares (in %)

Sector	average share 1948-1953	average share 2001-2005
Durables	17.86	8.39
NonDurables	15.04	6.14
FinInsur	3.27	9.19
ServScieTech	1.93	7.84
ServicesHealth	2.02	7.83

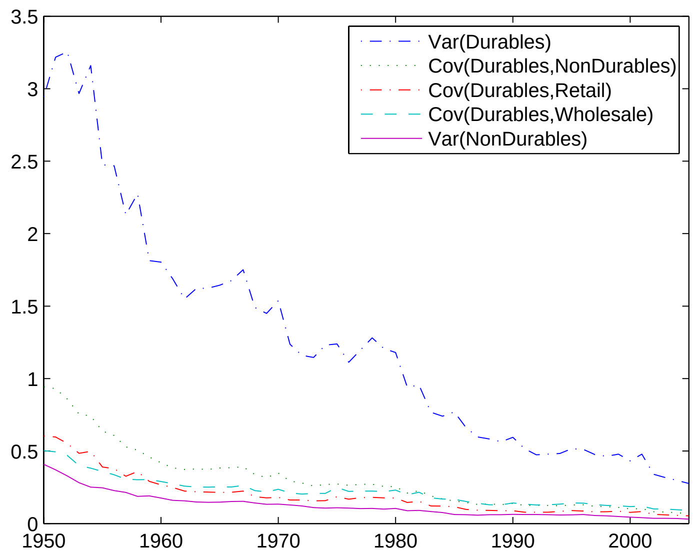
Figure 8: Conditional Standard Deviations



The above analysis does not take into account the evolution of covariance terms. Therefore, I now use another way to identify the relevant sectoral shifts. This way refers to equation (7). Concretely, I analyze the evolution of the terms $\tau^i_{t-1}\tau^k_{t-1}(W_t)_{ik}$ where $(W_t)_{ik}$ is the ikth element of the covariance matrix W_t and τ^i_{t-1} the share of sector i at time $t-1$. Hence, this analysis takes into consideration the path of the entire covariance matrices (W_t). My findings are the following. The $\tau^i_{t-1}\tau^k_{t-1}(W_t)_{ik}$-term, which decreased the most, corresponds to the variance of durable goods growth. Other terms,

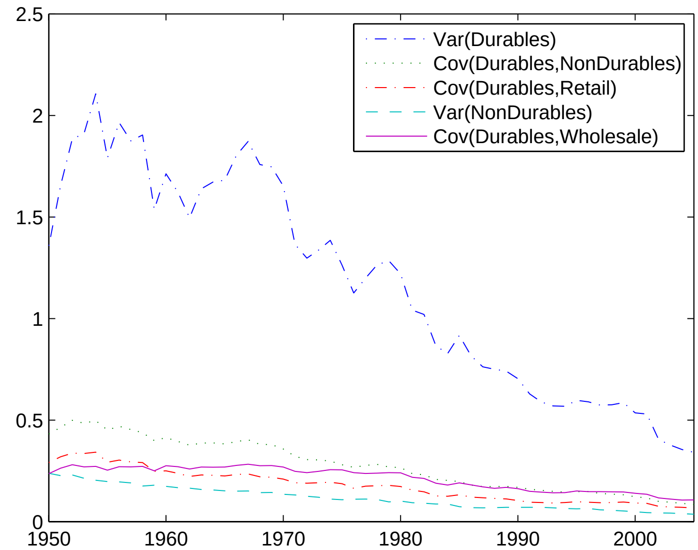
which decreased significantly, correspond - in order of descending magnitude - to:
(i) the covariance between durable and nondurable goods production,
(ii) the covariance between durable and retail sales,
(iii) the covariance between durable goods and wholesale trade and
(iv) the variance of nondurable goods production.
(These fives terms are plotted in Figure 9).

Figure 9: Five most important $\tau_{t-1}^{i}\tau_{t-1}^{k}(W_t)_{ik}$-terms



In order to disentangle the changes in the shifts from the changes in the covariance matrix, I also computed $\tau_{t-1}^{i}\tau_{t-1}^{k}(\overline{W})_{ik}$ where $\overline{W}$ is the average covariance matrix of the entire sample 1949-2005. Once more, the most important drop is found in the term corresponding to the variance of durable goods production. The other four significant drops are, in order of decreasing importance, the covariance between durable and nondurable goods, the covariance between durable goods and retail trade, the variance of nondurable goods and the covariance between durable goods and wholesale trade. These five $\tau_{t-1}^{i}\tau_{t-1}^{k}(\overline{W})_{ik}$-terms are plotted in Figure 10.

Figure 10: Five most important $\tau_{t-1}^{i}\tau_{t-1}^{k}(\overline{W})_{ik}$-terms



Based on these calculations, I conclude that the shift out of durable goods production has significantly stabilized real GDP growth. It has done so not only through a decreasing variance term, but also through decreasing covariance terms.

5 Conclusion

I analyzed how real GDP volatility is influenced by the composition of GDP and came to the conclusion that sectoral shifts had a non-negligible influence on output volatility.

Using simple Monte Carlo simulations, I found that the unconditional real GDP standard deviation would have dropped from 2.93 to 2.49 (-15%) in 1984, even if the covariance matrix of sectoral growth rates had not changed (i.e. had stayed equal to its pre-84 average). Hence, sectoral shifts alone would have produced a moderation of around 30% of the observed moderation.

Furthermore, I developed a new methodology - based on the particle filter and on the autoregressive Wishart process - to filter unobserved covariance matrices. With it, I expressed both, the conditional and unconditional standard deviation GDP growth as a function of the composition of GDP. This allowed me to show that if in the year 1949 sectoral shares had been equal to what they were in 2005 (1977), then the conditional standard deviation of GDP growth would have been, on average, 24.7% (19.1%) lower during the postwar period. Therefore, I argued that sectoral shifts made GDP easier to forecast. Concerning the unconditional standard deviation, I found that it would have been 25.6% (17.7%) lower in the postwar period had the composition of GDP been already in 1949 what it was in 2005 (1977).

I further found that an important component of the increased stability in real GDP growth was the shift out of durable goods production. This extends the findings of McConnell and Perez-Quiros (2000) who argue that the reduction in the variance of durable goods production alone can account of the break in GDP volatility.

Moreover, I gave evidence against the good luck hypothesis using the cumulative sums of squares test of Inclán and Tiao (1994): I found that only the conditional variance of durable goods production experienced a significant drop whereas the conditional variances of mining, utilities and information increased during the postwar period.

Finally, I showed that the hypothesis of a relationship between GDP composition and its volatility is not in contradiction with volatility inference made from a univariate ARCH model of GDP growth. This contributes to the discussion launched by Fang and Miller (2006) and Blanchard and Simon (2001) whether GDP volatility experienced a one time drop or a trend decrease.

Many of the existing economic models, which focus on business cycles, shut out the growth component. However, as I have shown, this leaves out an important part of the story. Therefore, future research should aim at developing behavioral models which rationalize the observed sectoral shifts, thus linking growth and volatility aspects.

Promising sources of inspiration could be Bernanke (1983) and Pindyck (1991). They show that if investment is irreversible, it can become sensitive to risk. Uncertainty can push the investor to postpone projects and wait for new information about prices, costs and other market conditions. This leads to a negative relationship between growth and volatility. Martin and Rogers (1987) argue that if human capital growth is an increasing and concave function in the cyclical component of output, then should GDP growth be negatively correlated with its volatility. Blackburn and Galindev (2003) show that if growth is mainly fostered by internal learning (people within the firm reduce production in order to have time to learn how to improve the productivity), then the correlation between growth and its volatility is more likely to be positive whereas it is more likely to be negative when external learning (meaning that aggregate labor supply has a positive spill-over effect on productivity growth) is more important. This emerges because the endogenous productivity growth rate is an increasing and convex function in the sole shock (a demand shock) when internal learning is more important whereas it is an increasing and concave function in predominance of external learning.

One of my conjectures is that increased openness allowed the US economy to export durable goods production. This would be consistent with (Barrell and Gottschalk, 2004) who find that changes in openness are important in explaining the decline in US-output volatility. Developing a model which incorporates all this is a challenging task I intend to go about in future research.

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A Appendix

A.1 Quarterly GDP with GARCH(1,1) and ARCH(2) Specifications

Table 15: **Linear Trend in Volatility**

GARCH Results

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2
0.4819***	0.2687***	0.1630**
(0.0782)	(0.0719)	(0.0720)

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \omega\ t + \gamma_1\ d_t + \gamma_2\ d_t^{70s}$

α_0	α_1	β_1	ω	γ_1	γ_2
0.4884*	0.1218	0.4882*	−0.0004	−0.3207*	0.2504
(0.2955)	(0.0837)	(0.2749)	(0.0008)	(0.1980)	(0.2644)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
2.2515	5.7859	0.5587	3.9173	0.6548
[0.522]	[0.448]	[0.906]	[0.688]	[0.721]

ARCH Results

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2
0.4743***	0.2827***	0.1566**
(0.0791)	(0.0748)	(0.0690)

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 + \omega\ t + \gamma_1\ d_t + \gamma_2\ d_t^{70s}$

α_0	α_1	α_1	ω	γ_1	γ_2
0.9436***	0.2045**	0.0581	-0.0011	−0.5611**	0.5296
(0.2183)	(0.0922)	(0.0964)	(0.0012)	(0.2663)	(0.3901)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
2.2147	5.8575	0.4828	3.6617	0.6449
[0.529]	[0.439]	[0.923]	[0.722]	[0.724]

Standard errors are in parentheses, p-values in brackets; $LBQ(k)$ and $LBQ^2(k)$ are the Ljung-Box Q-statistics for the standardised residuals and the squared standardised residuals testing for autocorrelation up to k lags. *, ** and *** denote 10, 5 and 1 % significance level.

Table 16: **GDP - Quadratic Trend in Volatility, Break 1984:1**

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2
0.4827***	0.2342***	0.1809***
(0.0762)	(0.0723)	(0.0647)

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \omega_1\ t + \omega_2\ t^2 + \gamma_1\ d_t + \gamma_2\ d_t^{70s}$

α_0	α_1	ω_1	ω_2	γ_1	γ_2
1.4729***	0.1348	0.0114*	0.00003*	−0.2702	0.8159**
(0.4061)	(0.0909)	(0.0071)	(0.00002)	(0.3872)	(0.4257)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
2.5148	5.0328	0.7492	3.9438	0.1138
[0.473]	[0.540]	[0.862]	[0.684]	[0.945]

Standard errors are in parentheses, p-values in brackets; $LBQ(k)$ and $LBQ^2(k)$ are the Ljung-Box Q-statistics for the standardised residuals and the squared standardised residuals testing for autocorrelation up to k lags. *, ** and *** denote 10, 5 and 1 % significance level.

Table 17: **Quadratic Trend in Volatility**

GARCH Results

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2	λ
0.4838***	0.2464***	0.1837***	
(0.0765)	(0.0684)	(0.0696)	

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \omega_1\ t + \omega_2\ t^2 + \gamma_1\ d_t + \gamma_2\ d_t^{70s}$

α_0	α_1	β_1	ω_1	ω_2	γ_1	γ_2
0.7526***	0.1214	0.4353**	−0.0061***	0.00002***	−0.0885	0.5197**
(0.2731)	(0.0980)	(0.2829)	(0.0006)	(0.000003)	(0.2344)	(0.3812)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
1.9575	5.4704	1.0291	3.7182	0.3554
[0.581]	[0.485]	[0.794]	[0.715]	[0.837]

ARCH Results

Growth rate equation: $y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t$

a_0	a_1	a_2	λ
0.4547***	0.2897***	0.1767**	
(0.0728)	(0.0697)	(0.0631)	

Volatility equation: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 + \omega_1\ t + \omega_2\ t^2 + \gamma_1\ d_t + \gamma_2\ d_t^{70s}$

α_0	α_1	α_1	ω_1	ω_2	γ_1	γ_2
1.2525***	0.1748**	0.02475	−0.0079***	0.00002***	−0.2374	0.5903**
(0.1947)	(0.0849)	(0.0714)	(0.0005)	(0.000003)	(0.2278)	(0.2915)

Test statistics

$LBQ(3)$	$LBQ(6)$	$LBQ^2(3)$	$LBQ^2(6)$	Jarque-Bera
1.8534	5.2437	0.8888	3.7556	0.486
[0.603]	[0.513]	[0.828]	[0.710]	[0.784]

Standard errors are in parentheses, p-values in brackets; $LBQ(k)$ and $LBQ^2(k)$ are the Ljung-Box Q-statistics for the standardised residuals and the squared standardised residuals testing for autocorrelation up to k lags. *, ** and *** denote 10, 5 and 1 % significance level.

A.2 Simple Two Period Monte Carlo Simulation

Table 18: With Agriculture and Federal Government

Simulation 1: Different Covariances in Period 1 and Period 2

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
2.87 (2.85)	$[2.16, 3.66]$	1.77 (1.65)	$[1.04, 3.33]$

Simulation 2: Covariances of Period 1 in both Periods

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
2.87 (2.85)	$[2.16, 3.66]$	2.50 (2.46)	$[1.62, 3.62]$

Simulation 3: Covariances of Period 2 in both Periods

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
2.00 (1.95)	$[1.47, 2.87]$	1.85 (1.66)	$[1.07, 4.22]$

Table 19: Constant AR(1) Coefficients

Simulation 1: Different Covariances in Period 1 and Period 2

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
2.87 (2.85)	[2.17, 3.66]	1.77 (1.65)	[1.05, 3.34]

Simulation 2: Covariances of Period 1 in both Periods

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
2.87 (2.85)	[2.16, 3.66]	2.49 (2.45)	[1.62, 3.61]

Simulation 3: Covariances of Period 2 in both Periods

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
2.01 (1.95)	[1.47, 2.87]	1.77 (1.65)	[1.05, 3.34]

Table 20: Series without breaks and remaining correlation

Simulation 1: Different Covariances in Period 1 and Period 2

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
4.13 (4.11)	$[3.14, 5.22]$	2.18 (2.16)	$[1.47, 3.04]$

Simulation 2: Covariances of Period 1 in both Periods

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
4.13 (4.11)	$[3.14, 5.22]$	3.75 (3.71)	$[2.54, 5.18]$

Simulation 3: Covariances of Period 2 in both Periods

Period 1		Period 2	
Average (Median)	95% Interval	Average (Median)	95% Interval
2.33 (2.32)	$[1.77, 2.93]$	2.20 (2.18)	$[1.50, 3.02]$

A.3 Filtered Standard Deviations Using a Wishart Prior

In order to save space, only the filtered standard deviations for mining, utilities, durable goods and information are reported. All other graphs are available upon request. For these four sectors, the ICSS test detected breaks in the conditional variance (see Table 9) which is reflected in the below graphs.

Figure 11: Mining

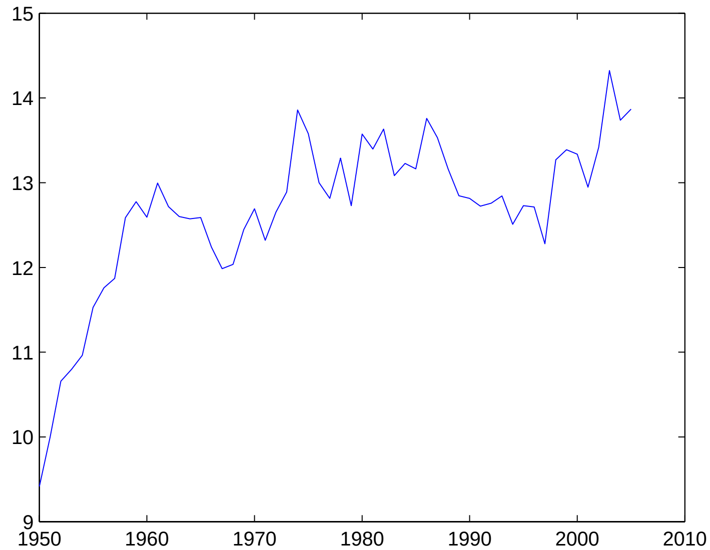


Figure 12: Utilities

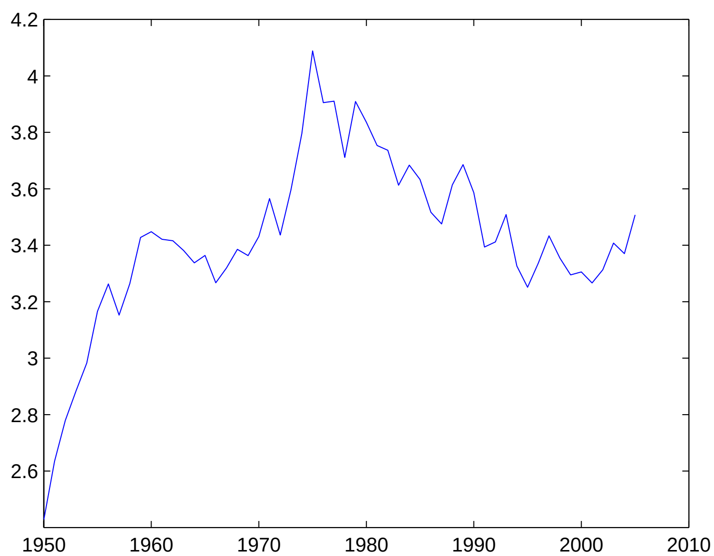


Figure 13: Durables

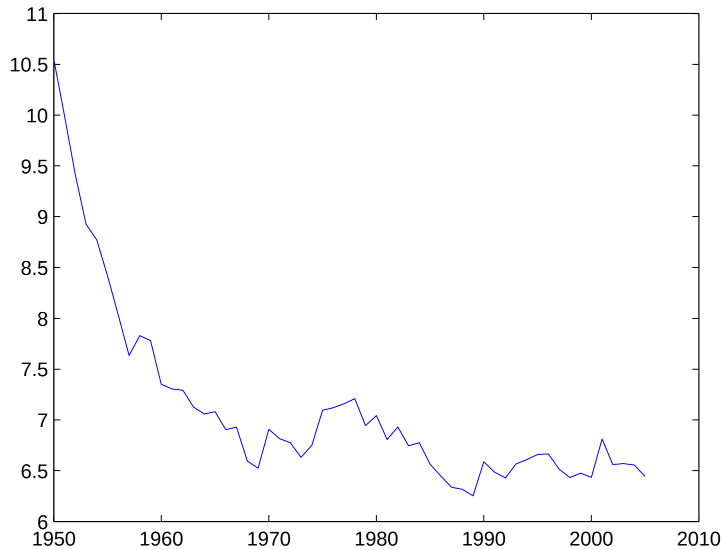
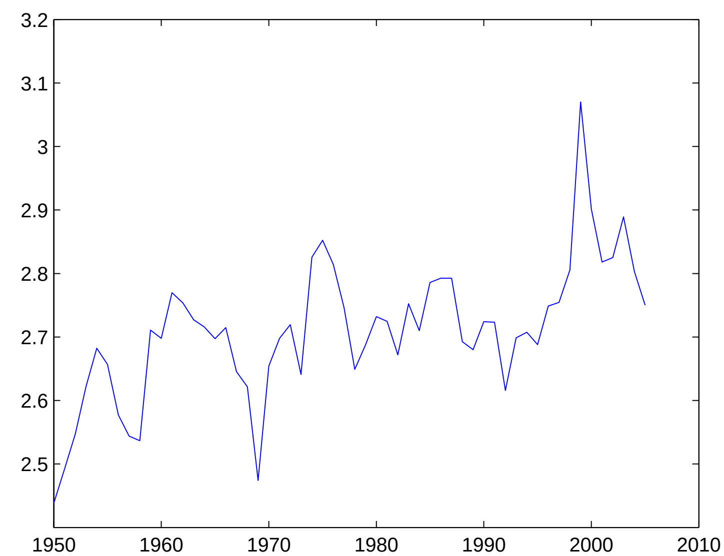
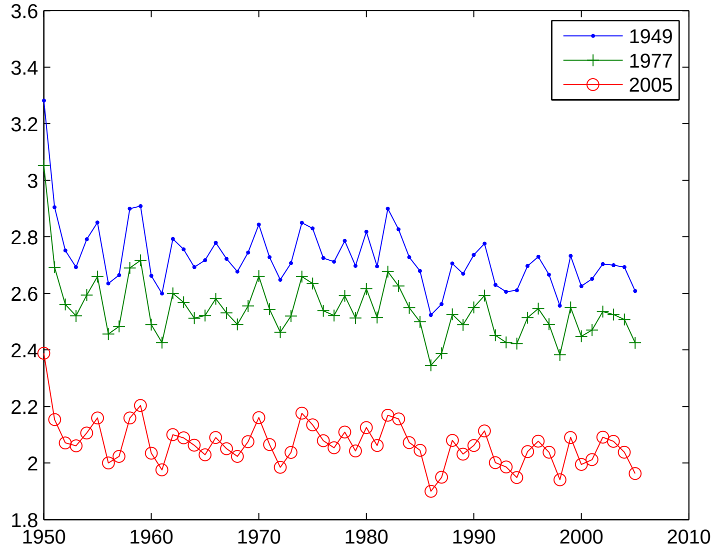


Figure 14: Information



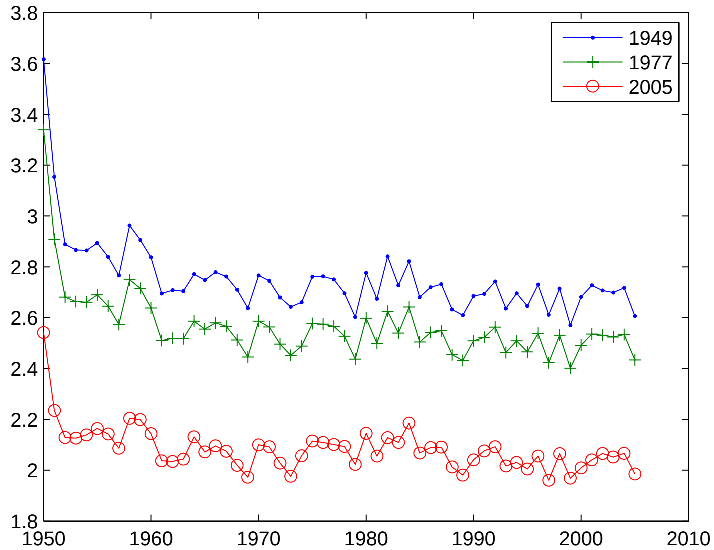
A.4 Alternative Wishart Specifications

Figure 15: Conditional GDP Standard Deviation with given constant shares and $\lambda = 0.2$



On average, the conditional standard deviations with 2005-shares are 24.3% (18.7%) lower than the ones with 1949-shares (1977-shares).

Figure 16: Conditional GDP Standard Deviation with given constant shares and $\lambda = 0.5$



On average, the conditional standard deviations with 2005-shares are 24.3% (18.8%) lower than the ones with 1949-shares (1977-shares).